EN475 – OCEAN ENGINEERING MECHANICS – *Equation Sheet 1:* **Linear Wave Properties**

	Shallow Water	***Intermediate Depth***	***Deep Water***
Surface Elev., h	$h = \frac{H}{2} \cos(kx - st)$	$h = \frac{H}{2} \cos(kx - st)$	$h = \frac{H}{2} \cos(kx - st)$
Wave Length, L	$L = T\sqrt{gd}$	$L = \frac{gT^2}{2p} \tanh(kd)$ $L = L_0 \tanh(kd)$	$L_0 = \frac{gT^2}{2p}$
Wave Speed or Celerity, C=L/T	$C = \sqrt{gd}$	$C = \sqrt{\frac{g}{k} \tanh kd} = \frac{gT}{2p} \tanh kd$	$C_0 = \sqrt{\frac{g}{k_0}} = \frac{gT}{2p}$
Potential, f	$f = \frac{Hg}{2s} \sin(kx - st)$	$f = \frac{Hg}{2s} \frac{\cosh k(z+d)}{\cosh kd} \sin(kx - st)$	$f = \frac{Hg}{2s} e^{kz} \sin(kx - st)$
Horizontal (x-dir'n) velocity, u	$u = \frac{Hgk}{2s} \cos(kx - st)$ $u = \frac{H}{2}\sqrt{\frac{g}{d}} \cos(kx - st)$ $u = \frac{pH}{T}\frac{1}{kd} \cos(kx - st)$	$u = \frac{Hgk}{2s} \frac{\cosh k(z+d)}{\cosh kd} \cos(kx - st)$ $u = \frac{pH}{T} \frac{\cosh k(z+d)}{\sinh kd} \cos(kx - st)$ $u = \frac{pH}{T} K_x \cos(kx - st)$	$u = \frac{Hgk}{2s} e^{kz} \cos(kx - st)$ $u = \frac{pH}{T} e^{kz} \cos(kx - st)$
Vertical (z-dir'n) velocity, w	$w = \frac{Hg(z+d)}{2sd} \sin(kx - st)$ $w = \frac{pH}{T}\left(1 + \frac{z}{d}\right) \sin(kx - st)$	$w = \frac{Hgk}{2s} \frac{\sinh k(z+d)}{\cosh kd} \sin(kx - st)$ $w = \frac{pH}{T} \frac{\sinh k(z+d)}{\sinh kd} \sin(kx - st)$ $w = \frac{pH}{T} K_z \sin(kx - st)$	$w = \frac{Hgk}{2s} e^{kz} \sin(kx - st)$ $w = \frac{pH}{T} e^{kz} \sin(kx - st)$
Horizontal Particle Acceleration, a_x	$a_x = \frac{Hg}{2d} \sin(kx - st)$	$a_x = \frac{2p^2 H}{T^2} K_x \sin(kx - st)$	$a_x = \frac{Hgk}{2} e^{kz} \sin(kx - st)$
Vertical Particle Acceleration, a_z	$a_z = -\frac{Hg(z+d)}{2d} \cos(kx - st)$	$a_z = -\frac{2p^2 H}{T^2} K_z \cos(kx - st)$	$a_z = -\frac{Hgk}{2} e^{kz} \cos(kx - st)$
Particle Paths	tend to lines	elliptical	circular

	Shallow Water	***Intermediate Depth***	***Deep Water***
Pressure, $p_{total} = p_{static} + p_{dynamic}$	$p_{total} = -r\,g\,z + r\,g\,\frac{H}{2}\cos(k\,x - s\,t)$ $p_{total} = r\,g\,(h - z)$	$p_{total} = -r\,g\,z + r\,g\,\frac{H}{2}\,\frac{\cosh k(z+d)}{\cosh kd}\cos(k\,x - s\,t)$ $p_{total} = -r\,g\,z + r\,g\,\frac{H}{2}\,K_p\cos(k\,x - s\,t)$ $p_{total} = r\,g\,(K_p h - z)$	$p_{total} = -r\,g\,z + r\,g\,\frac{H}{2}\,e^{kz}\cos(k\,x -$ $p_{total} = r\,g\,(e^{kz} h - z)$
Energy in One Wave Length, E	$E = \frac{r\,g\,H^2 L}{8}$	$E = \frac{r\,g\,H^2 L}{8}$	$E = \frac{r\,g\,H^2 L}{8}$
Average Energy Density, $\overline{E}$	$\overline{E} = \frac{r\,g\,H^2}{8}$	$\overline{E} = \frac{r\,g\,H^2}{8}$	$\overline{E} = \frac{r\,g\,H^2}{8}$
Power, $P = \frac{n\,E}{T}$	$P = \overline{E}\,C = \overline{E}\,C_g$	$P = \frac{\overline{E}\,C}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)$ $P = n\,\overline{E}\,C = \overline{E}\,C_g$	$P = \frac{\overline{E}\,C}{2} = \overline{E}\,C_g$
Group Velocity, C_g	$C_g = C$	$C_g = \frac{C}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right)$ $C_g = n\,C$	$C_g = \frac{C}{2}$
Dispersion Equation	$s^2 = k^2 g\,d$	$s^2 = g\,k\,\tanh k\,d$	$s^2 = g\,k$

Shoaling & Refraction: $H = H_0 K_S K_R$ $\quad K_S = \sqrt{\frac{C_{g_0}}{C_g}}$ $\quad K_R = \sqrt{\frac{\cos a_0}{\cos a}}$ $\quad a = \sin^{-1}\left(\frac{C}{C_0}\sin a_0\right) = \sin^{-1}\left(\tanh kd\,\sin a_0\right)$

Standing Waves: surface elevations: $h_i = \frac{H}{2}\cos(kx - st)$ $\quad h_r = \frac{H}{2}\cos(kx + st)$ $\quad h = h_i + h_r = H_i\cos(kx)\cos(st)$

dynamic pressure: $p_d = p_{d_i} + p_{d_r} = rg\,H_i\,\frac{\cosh k(z+d)}{\cosh kd}\cos(kx)\cos(st)$

velocities: $u = u_i + u_r = \frac{2p\,H_i}{T}\,\frac{\cosh k(z+d)}{\sinh kd}\sin(kx)\sin(st)$ $\quad w = w_i + w_r = \frac{2p\,H_i}{T}\,\frac{\sinh k(z+d)}{\sinh kd}\cos(kx)\sin(st)$

H = wave height $\quad s = \frac{2p}{T}$ $\quad k = \frac{2p}{L}$ $\quad$ *T = wave period* $\quad$ *L = wave length* $\quad$ *d = water depth* $\quad K_p = \frac{\cosh k(z+d)}{\cosh kd}$ $\quad K_x = \frac{\cosh k(z+d)}{\sinh kd}$ $\quad K_z = \frac{\sinh k(z+d)}{\sinh kd}$ $\quad n = \frac{1}{2}\left[1 + \frac{2kd}{\sinh 2kd}\right]$